

WHITE PAPER

# Interest Rate Sensitivity

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#### Introduction

The interest rate sensitivity widget and report estimate how sensitive a portfolio's fixed income holdings are to changes in interest rates. Interest rate sensitivity is a measure of how much the price of a fixed income holding will fluctuate due to changes in the interest rate environment. Holdings that are more sensitive may have greater price fluctuations than those with less sensitivity.

#### **Bond Example**

For each calculation, a hypothetical bond with the below characteristics will be used.

Face Value: \$1,000 Current Market Price: \$973.3569

Coupon Rate: 6% paid semi-annually (\$30 per payment)

Current YTM: 7%

Maturity: 3 years (6 semi-annual periods)

Issue Date: January 15, 2023 Maturity Date: January 15, 2026

## **Macaulay Duration**

Measures in years the weighted average time until a bond's cash flows are received, reflecting the bond's sensitivity to interest rate changes. Macaulay duration quantifies the time it takes for an investor to be repaid through a combination of interest and principal payments.

# **Macaulay Duration Formula**

$$Macaulay Duration = \frac{\sum_{i=1}^{n} PV(i) \times \frac{t \times CF(i)}{P}}{P}$$

#### Where:

PV = Present value of each cash flow =  $CF/(1+r)^t$ t = Time period of each cash flow (in years)

CF = Cash flow amount (coupon payment or coupon + face value)

P = Current bond price

r = Yield to maturity per period (YTM/2 for semi-annual bonds)

n = Total number of periods



## **Macaulay Duration Example**

$$i = 1: [(30/1.035^1) \times (0.5/973.3569)] \times 973.3569 = 14.492$$

$$i = 2: [(30/1.035^2) \times (1.0/973.3569)] \times 973.3569 = 28.0053$$

$$i = 3: [(30/1.035^3) \times (1.5/973.3569)] \times 973.3569 = 40.5874$$

$$i = 4: [(30/1.035^4) \times (2.0/973.3569)] \times 973.3569 = 52.2865$$

$$i = 5: [(30/1.035^5) \times (2.5/973.3569)] \times 973.3569 = 63.1479$$

$$i = 6: [(1030/1.035^6) \times (3.0/973.3569)] \times 973.3569 = 2,513.7169$$

Macaulay Duration = Sum of all terms/P = 2,712.2367/973.3569 = 2.7865 years

# **Modified Duration**

Measures the percentage change in a bond's price for a 1% (100 basis point) change in yield. It represents the slope of the price-yield relationship at a given point.

#### **Modified Duration Formula**

$$Modified Duration = \frac{Macaulay Duration}{1 + \left(\frac{YTM}{n}\right)}$$

Where:

YTM = Yield to maturity

n = Total number of coupon periods per year

# **Modified Duration Example**

Modified Duration = 
$$\frac{2.7865}{1 + \left(\frac{0.07}{2}\right)} = \frac{2.7865}{1.035} = 2.6922$$



#### Convexity

It measures the curvature of the relationship between bond prices and yields. Convexity demonstrates why Modified Duration alone (which assumes a linear relationship) becomes less accurate for larger interest rate changes. Applying Convexity to the calculated Modified Duration makes price estimates more accurate, especially for larger yield changes.

# **Convexity Formula**

Convexity 
$$\approx \frac{P(+) + P(-) - 2P(0)}{P(0) \times \Delta y^2}$$

# Where:

P(+) = Bond market value after yield increase P(-) = Bond market value after yield decrease

P(0) = Current bond market value

 $\Delta y =$  Change in yield (typically 0.0001 or 1 basis point)

# **Convexity Example**

P(+) calculation using  $\Delta y = 0.01\%$  resulting in a current YTM of 3.51% ((7.0%/2) + 0.01%) per payment period.

$$P(+) = \frac{30}{1.0351^{1}} + \frac{30}{1.0351^{2}} + \frac{30}{1.0351^{3}} + \frac{30}{1.0351^{4}} + \frac{30}{1.0351^{5}} + \frac{1030}{1.0351^{6}}$$

$$P(+) = 28.9827 + 27.9999 + 27.0504 + 26.1331 + 25.2469 + 837.4200$$

$$P(+) = 972.833$$

P(-) calculation using  $\Delta y = -0.01\%$  resulting in a current YTM of 3.49% ((7.0%/2) -0.01%) per payment period.

$$P(-) = \frac{30}{1.0349^{1}} + \frac{30}{1.0349^{2}} + \frac{30}{1.0349^{3}} + \frac{30}{1.0349^{4}} + \frac{30}{1.0349^{5}} + \frac{1030}{1.0349^{6}}$$

$$P(-) = 28.9883 + 28.0107 + 27.0661 + 26.1533 + 25.2714 + 838.3915$$

$$P(-) = 973.8813$$

Calculating Convexity using the P(+) and P(-) values above.

Convexity 
$$\approx \frac{972.833 + 973.8813 - 2(973.3569)}{973.3569 \times 0.0001^2}$$

$$Convexity \approx \frac{0.0005}{0.00000973}$$

 $Convexity \approx 51.3874$ 



# **Estimating Bond Value Change**

For a given rate change, both the Duration Effect and Convexity Effect can be used to more accurately estimate how the value of a fixed income holding will be impacted.

# **Bond Value Change Formula**

$$Duration\ Effect = P(0)\ \times\ (Modified\ Duration\ \times\ -\Delta y)$$

Convexity Effect = 
$$P(0) \times \left(\frac{1}{2} \times Convexity \times \Delta y^2\right)$$

$$\Delta P = Duration \ Effect + Convexity \ Effect$$

# Where:

P(0) = Current bond market value  $\Delta y = Change in interest rate$  $\Delta P = Total bond value change$ 

# **Bond Value Change Example**

Current bond value P(0) is \$973.36. See below  $\Delta P$  calculation assuming a +2% rate change.

*Duration Effect* = 
$$973.3569 \times (2.6922 \times -0.02)$$

$$Duration\ Effect = -52.4094$$

Convexity Effect = 973.3569 
$$\times \left(\frac{1}{2} \times 51.3874 \times 0.02^2\right)$$

$${\it Convexity \, Effect} = 10.0036$$

$$\Delta P = -52.4094 + 10.0036$$

$$\Delta P = -42.4058 = -\$42.41$$

Rate Change	<b>Duration Effect</b>	Convexity Effect	ΔΡ	Est. Bond Value	% Value Change
+2%	-\$52.41	\$10.00	-\$42.41	\$930.95	-4.36%
0%	\$0.00	\$0.00	\$0.00	\$973.36	0.0%
-2%	\$52.41	\$10.00	\$62.41	\$1,035.77	6.41%



#### Conclusion

Within the Interest Rate Sensitivity report and widget, we calculate the Macaulay Duration for each fixed income holding in a portfolio. Using Macaulay Duration, we calculate the Modified Duration which is used to determine the Duration Effect from a given interest rate change. We then estimate each bond position's Convexity by calculating the bond value given a small positive and negative interest rate change. Using a bonds calculated Convexity allows us to determine the Convexity Effect for a given interest rate change. The total estimated bond value change for a given interest rate change is then calculated by combining the Duration Effect and the Convexity Effect demonstrating a position or portfolio's interest rate sensitivity.

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