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# Interest Rate Sensitivity

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## Introduction

The interest rate sensitivity widget and report estimate how sensitive a portfolio's fixed income holdings are to changes in interest rates. Interest rate sensitivity is a measure of how much the price of a fixed income holding will fluctuate due to changes in the interest rate environment. Holdings that are more sensitive may have greater price fluctuations than those with less sensitivity.

## Bond Example

For each calculation, a hypothetical bond with the below characteristics will be used.

Face Value:	\$1,000
Current Market Price:	\$973.3569
Coupon Rate:	6% paid semi-annually (\$30 per payment)
Current YTM:	7%
Maturity:	3 years (6 semi-annual periods)
Issue Date:	January 15, 2023
Maturity Date:	January 15, 2026

## Macaulay Duration

Measures in years the weighted average time until a bond's cash flows are received, reflecting the bond's sensitivity to interest rate changes. Macaulay duration quantifies the time it takes for an investor to be repaid through a combination of interest and principal payments.

## Macaulay Duration Formula

$$\text{Macaulay Duration} = \frac{\sum_{i=1}^n PV(i) \times \frac{t \times CF(i)}{P}}{P}$$

## Where:

PV =	Present value of each cash flow = $CF/(1+r)^t$
t =	Time period of each cash flow (in years)
CF =	Cash flow amount (coupon payment or coupon + face value)
P =	Current bond price
r =	Yield to maturity per period ( $YTM/2$ for semi-annual bonds)
n =	Total number of periods

### Macaulay Duration Example

$$i = 1: [(30/1.035^1) \times (0.5/973.3569)] \times 973.3569 = 14.492$$

$$i = 2: [(30/1.035^2) \times (1.0/973.3569)] \times 973.3569 = 28.0053$$

$$i = 3: [(30/1.035^3) \times (1.5/973.3569)] \times 973.3569 = 40.5874$$

$$i = 4: [(30/1.035^4) \times (2.0/973.3569)] \times 973.3569 = 52.2865$$

$$i = 5: [(30/1.035^5) \times (2.5/973.3569)] \times 973.3569 = 63.1479$$

$$i = 6: [(1030/1.035^6) \times (3.0/973.3569)] \times 973.3569 = 2,513.7169$$

$$\text{Macaulay Duration} = \text{Sum of all terms}/P = 2,712.2367/973.3569 = 2.7865 \text{ years}$$

### Modified Duration

Measures the percentage change in a bond's price for a 1% (100 basis point) change in yield. It represents the slope of the price-yield relationship at a given point.

### Modified Duration Formula

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{1 + \left(\frac{YTM}{n}\right)}$$

#### Where:

YTM = Yield to maturity

n = Total number of coupon periods per year

### Modified Duration Example

$$\text{Modified Duration} = \frac{2.7865}{1 + \left(\frac{0.07}{2}\right)} = \frac{2.7865}{1.035} = 2.6922$$

## Convexity

It measures the curvature of the relationship between bond prices and yields. Convexity demonstrates why Modified Duration alone (which assumes a linear relationship) becomes less accurate for larger interest rate changes. Applying Convexity to the calculated Modified Duration makes price estimates more accurate, especially for larger yield changes.

## Convexity Formula

$$\text{Convexity} \approx \frac{P(+)+P(-)-2P(0)}{P(0) \times \Delta y^2}$$

### Where:

P(+) = Bond market value after yield increase

P(-) = Bond market value after yield decrease

P(0) = Current bond market value

$\Delta y$  = Change in yield (typically 0.0001 or 1 basis point)

## Convexity Example

P(+) calculation using  $\Delta y = 0.01\%$  resulting in a current YTM of 3.51% ((7.0%/2) + 0.01%) per payment period.

$$P(+)=\frac{30}{1.0351^1}+\frac{30}{1.0351^2}+\frac{30}{1.0351^3}+\frac{30}{1.0351^4}+\frac{30}{1.0351^5}+\frac{1030}{1.0351^6}$$

$$P(+)=28.9827+27.9999+27.0504+26.1331+25.2469+837.4200$$

$$P(+)=972.833$$

P(-) calculation using  $\Delta y = -0.01\%$  resulting in a current YTM of 3.49% ((7.0%/2) - 0.01%) per payment period.

$$P(-)=\frac{30}{1.0349^1}+\frac{30}{1.0349^2}+\frac{30}{1.0349^3}+\frac{30}{1.0349^4}+\frac{30}{1.0349^5}+\frac{1030}{1.0349^6}$$

$$P(-)=28.9883+28.0107+27.0661+26.1533+25.2714+838.3915$$

$$P(-)=973.8813$$

Calculating Convexity using the P(+) and P(-) values above.

$$\text{Convexity} \approx \frac{972.833+973.8813-2(973.3569)}{973.3569 \times 0.0001^2}$$

$$\text{Convexity} \approx \frac{0.0005}{0.00000973}$$

$$\text{Convexity} \approx 51.3874$$

## Estimating Bond Value Change

For a given rate change, both the Duration Effect and Convexity Effect can be used to more accurately estimate how the value of a fixed income holding will be impacted.

### Bond Value Change Formula

$$\text{Duration Effect} = P(0) \times (\text{Modified Duration} \times -\Delta y)$$

$$\text{Convexity Effect} = P(0) \times \left( \frac{1}{2} \times \text{Convexity} \times \Delta y^2 \right)$$

$$\Delta P = \text{Duration Effect} + \text{Convexity Effect}$$

#### Where:

$P(0)$  = Current bond market value

$\Delta y$  = Change in interest rate

$\Delta P$  = Total bond value change

### Bond Value Change Example

Current bond value  $P(0)$  is \$973.36. See below  $\Delta P$  calculation assuming a +2% rate change.

$$\text{Duration Effect} = 973.3569 \times (2.6922 \times -0.02)$$

$$\text{Duration Effect} = -52.4094$$

$$\text{Convexity Effect} = 973.3569 \times \left( \frac{1}{2} \times 51.3874 \times 0.02^2 \right)$$

$$\text{Convexity Effect} = 10.0036$$

$$\Delta P = -52.4094 + 10.0036$$

$$\Delta P = -42.4058 = -\$42.41$$

Rate Change	Duration Effect	Convexity Effect	$\Delta P$	Est. Bond Value	% Value Change
+2%	-\$52.41	\$10.00	-\$42.41	\$930.95	-4.36%
0%	\$0.00	\$0.00	\$0.00	\$973.36	0.0%
-2%	\$52.41	\$10.00	\$62.41	\$1,035.77	6.41%

## Conclusion

Within the Interest Rate Sensitivity report and widget, we calculate the Macaulay Duration for each fixed income holding in a portfolio. Using Macaulay Duration, we calculate the Modified Duration which is used to determine the Duration Effect from a given interest rate change. We then estimate each bond position's Convexity by calculating the bond value given a small positive and negative interest rate change. Using a bonds calculated Convexity allows us to determine the Convexity Effect for a given interest rate change. The total estimated bond value change for a given interest rate change is then calculated by combining the Duration Effect and the Convexity Effect demonstrating a position or portfolio's interest rate sensitivity.

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